Is $D_{sJ}^+(2632)$ the first radial excitation of $D_s^*(2112)$?

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Abstract

We present a quantitative analysis of the $D_{sJ}^+(2632)$ observed by SELEX mainly focusing on the assumption that $D_{sJ}^+(2632)$ is the first radial excitation of the 1^- ground state $D_s^*(2112)$. By solving the instantaneous Bethe-Salpeter equation, we obtain the mass 2658 ± 15 MeV for the first excited state, which is about 26 MeV heavier than the experimental value 2632 ± 1.7 MeV. By means of PCAC and low-energy theorem we calculate the transition matrix elements and obtain the decay widths: $\Gamma(D_{sJ}^+ \to D_s^+ \eta) = 4.07 \pm 0.34$ MeV, $\Gamma(D_{sJ}^+ \to D^0 K^+) \simeq \Gamma(\Gamma(D_{sJ}^+ \to D^+ K^0) = 8.9 \pm 1.2$ MeV, and the ratio $\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+ \eta) = 2.2 \pm 0.2$ as well. This ratio is quite different from the SELEX data 0.14 ± 0.06. The summed decay width of those three channels is approximately 21.7 MeV, already larger than the observed bound for the full width (≤ 17 MeV). Furthermore, assuming $D_{sJ}^+(2632)$ is 1^- state, we also explore the possibility of $S - D$ wave mixing to explain the SELEX observation. Based on our analysis, we suspect that it is too early to conclude that $D_{sJ}^+(2632)$ is the first radial excitation of the 1^- ground state $D_s^*(2112)$. More precise measurements of the relative ratios and the total decay width are urgently required especially for $S - D$ wave mixing.

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I. INTRODUCTION

Last year the SELEX Collaboration reported the first observation of a narrow charmed meson $D_{sJ}^+(2632)$ with the mass of 2632.5 ± 1.7 MeV and decay width $\Gamma_{tot} \leq 17$ MeV at 90% confidence level [1]. This state has been seen in two decay modes, $D_s^+\eta$ and $D^0 K^+$, with a relative branching ratio $\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+\eta) = 0.14 \pm 0.06$. This relative branching ratio is rather unusual because the phase space of channel $D^0 K^+$ is 1.5 times larger than that of the channel $D_s^+\eta$ and according to the SU(3) flavor symmetry the ratio $\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+\eta) \geq 2.3$ [2] if the quark content of $D_{sJ}^+(2632)$ is $c\bar{s}$, and the reference [3] gave an estimate of $\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+\eta) = 7.0$. Since the $D_{sJ}^+(2632)$ is above the threshold for $D^0 K^+$ and $D_s^+\eta$, and its decays to these final states are Okubo-Zweig-Iizuka (OZI) allowed processes, it is also strange that it has a narrow width (≤ 17 MeV).

Why this state is so narrow and dominated by the $D_s^+\eta$ decay mode has inspired a lot of theoretical interest [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], with many possible explanations, *e.g.* the first radial excitation of $D_s^\star(2112)$, a tetraquark, a hybrid, a two-meson molecule, a diquark-antidiquark bound state, *etc.* Among them the most plausible candidate is the first radial excitation of $D_s^\star(2112)$, the 2^3S_1 state, since it is the likely conventional meson with component of $c\bar{s}$ which has not been found. And even though the mass of 2632 MeV is about 100 MeV below the traditional potential model prediction (for example, see [16]), the similar situation has been found in the new narrow charm-strange 0^+ meson $D_{sJ}(2317)$, favored as the conventional $c\bar{s}$ state. Furthermore, Simonov and Tjon [4] showed a possible mechanism to shift down the meson mass by the coupled-channel analysis method. Chao [5] gave a reasonable argument that the node structure in the 2^3S_1 state may explain the small decay width and unusual decay modes.

In this letter, following the references [3, 5], we assume $D_{sJ}^+(2632)$ as the first radial excitation of vector meson $D_s^*(2112)$. Then we solve the instantaneous Bethe-Salpeter equation [17], and give our prediction of mass for 2^3S_1 state. At the same time, we obtain the wave functions of the relevant mesons. By using the reduction formula, PCAC relation and low energy theorem, we write the transition S-matrix as a formula involving the light meson decay constant and the corresponding axial current transition matrix element between two heavy mesons, which in turn can be written as an overlapping integral of the relevant wave functions. We give a detailed consideration on the node structure in 2^3S_1 wave function. We further try to make a rough estimate on the full decay width and the ratio

$\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+ \eta)$ by assuming $D_{sJ}^+(2632)$ as the D-wave 1^- vector state as well as considering the possible $S - D$ wave mixing.

II.   PREDICTION OF MASS

In our previous papers [18, 19], we solved exactly the instantaneous Bethe-Salpeter equations (or the Salpeter equations [20]) to describe the behavior of 0^- and 1^- states. In Refs. [18], we have given the mass spectra and wave functions of pseudoscalar D_s, D^0 and D^+, and the mass spectra of all these heavy 0^- states, which are consistent with experimental data quite well. For the instantaneous Bethe-Salpeter equation of 1^- states [19], we have used the same parameters as used in [18], only changing the parameter V_0 to fit data because V_0 is the only parameter which can be different for 0^- and 1^- states when their quark contents are the same. In this letter we first solve the full Salpeter 1^- equation through fitting the ground state mass at 2112 MeV ($D_s^*(2112)$) to fix the parameter V_0. After fixing the parameter V_0, we then give the predictions of the first radial excitation state. Since we are only interested in the leading order estimate, which lets us see the problem clearly, we assume in our calculation that the first radial excitation state D_{sJ}^+ is a pure S wave state, excluding the possible mixing between S wave and D wave states. Later in this letter we will also consider the D wave contribution. Our prediction for the mass of the first radial excitation state is 2658 ± 15 MeV, which is about 26 MeV heavier than the experimental data 2632 ± 1.7 MeV.

III.   TRANSITION MATRIX ELEMENT

In this section we give a detailed description for the calculation of the transition matrix element. By using the reduction formula, the transition S-matrix of decay $D_{sJ}^+ \to D^0 K^+$ (similar to the channel $D_{sJ}^+ \to D^+ K^0$) can be written as:

$$\langle D(P_{f1}) K(P_{f2}) | D_{sJ}^+(P) \rangle = \int d^4 x e^{i P_{f2} \cdot x} (M_K^2 - P_{f2}^2) \langle D(P_{f1}) | \Phi_K(x) | D_{sJ}^+(P) \rangle, \qquad (1)$$

where P, P_{f1} and P_{f2} are the total momenta of initial state $2^3 S_1$, final state heavy meson and final state light meson, respectively (see Figure 1); M_K is the mass of the final light meson. The PCAC relates the meson field $\Phi_K(x)$ with a current $\Phi_K(x) = \frac{1}{M_K^2 f_K} \partial^\mu (\bar{q} \gamma_\mu \gamma_5 s)$, where f_K is the decay constant of light meson; q is u for K^+, d for K^0, respectively. Then
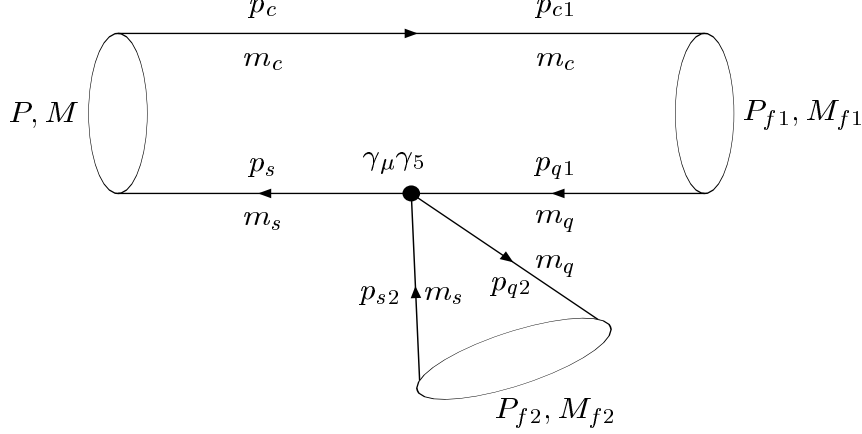
FIG. 1: Diagram for the transition of $D_{sJ}^+(2632)$ decay.

the S-matrix becomes

$$\langle D(P_{f1})K(P_{f2})|D_{sJ}^+(P)\rangle = \frac{(M_K^2 - P_{f2}^2)}{M_K^2 f_K} \int d^4x e^{iP_{f2}\cdot x} \langle D(P_{f1})|\partial^\mu(\bar{q}\gamma_\mu\gamma_5 s)|D_{sJ}^+(P)\rangle$$

$$= \frac{-iP_{f2}^\mu(M_K^2 - P_{f2}^2)}{M_K^2 f_K} \int d^4x e^{iP_{f2}\cdot x} \langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P)\rangle, \tag{2}$$

and with the low energy theorem this equation can be written approximately as:

$$\approx \frac{-iP_{f2}^\mu}{f_K} \int d^4x e^{iP_{f2}\cdot x} \langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle. \tag{3}$$

And, finally, we have

$$\langle D(P_{f1})K(P_{f2})|D_{sJ}^+(P_i)\rangle = (2\pi)^4\delta^4(P_i - P_{f1} - P_{f2})\frac{-iP_{f2}^\mu}{f_K}\langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle. \tag{4}$$

For the decay $D_{sJ}^+ \to D_s^+\eta$, the equation is similar, but there is $\eta - \eta'$ mixing. Including the $\eta - \eta'$ mixing, the PCAC relation reads as

$$\Phi_\eta(x) = \cos\theta\Phi_{\eta_8}(x) + \sin\theta\Phi_{\eta_0}(x)$$

$$= \frac{\cos\theta}{M_{\eta_8}^2 f_{\eta_8}}\partial^\mu(\frac{\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s}{\sqrt{6}}) + \frac{\sin\theta}{M_{\eta_0}^2 f_{\eta_0}}\partial^\mu(\frac{\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s}{\sqrt{3}})$$

$$= \partial^\mu(\bar{s}\gamma_\mu\gamma_5 s)\left[\frac{-2\cos\theta}{\sqrt{6}M_{\eta_8}^2 f_{\eta_8}} + \frac{\sin\theta}{\sqrt{3}M_{\eta_0}^2 f_{\eta_0}}\right], \tag{5}$$

where f_{η_8}, f_{η_0} are the decay constants of octet η_8 and singlet η_0, respectively. Then the transition matrix element becomes

$$\langle D_s(P_{f1})\eta(P_{f2})|D_{sJ}^+(P_i)\rangle = (2\pi)^4\delta^4(P_i - P_{f1} - P_{f2})(-iP_{f2}^\mu)$$

$$\times \left[\frac{-2M_\eta^2 \cos\theta}{\sqrt{6}M_{\eta_8}^2 f_{\eta_8}} + \frac{M_\eta^2 \sin\theta}{\sqrt{3}M_{\eta_0}^2 f_{\eta_0}} \right] \langle D_s(P_{f1})|\bar{s}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle. \tag{6}$$

Note that the decays $D_{sJ}^+(2632) \to D_s^+$ are OZI-rule-allowed, so we may ignore anomaly of the PCAC safely in Eqs. (2,6).

According to the Mandelstam formalism [21], at the leading order, the matrix element $\langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle$ can be written as [22]:

$$\langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P)\rangle = \int \frac{d\vec{q}}{(2\pi)^3} Tr\left[\bar{\varphi}_{P_{f1}}^{++}(\vec{q} - \frac{m_c}{m_c + m_q}\vec{r}) \frac{\not{P}}{M} \varphi_{P_i}^{++}(\vec{q})\gamma_\mu\gamma_5 \right], \tag{7}$$

where m_c and m_q is the mass of c and q ($q = u, d, s$) quark; $\vec{r}$ is the recoil three dimensional momentum of the final state D meson. p_c, p_s are the momenta of c and s quark in the initial meson, respectively. And the relative momentum $\vec{q}$ is defined as $\vec{q} \equiv \vec{p}_c - \frac{m_c}{m_c + m_s}\vec{P} \equiv \frac{m_s}{m_c + m_s}\vec{P} - \vec{p}_s$, in the center of mass system of the initial meson, and it becomes $\vec{q} \equiv \vec{p}_c \equiv -\vec{p}_s$; $\varphi_{P_i}^{++}$, and $\varphi_{P_{f1}}^{++}$ are called the positive energy wave function of the initial and final heavy mesons, and $\bar{\varphi}_{P_{f1}}^{++} = -\gamma_0(\varphi_{P_{f1}}^{++})^+\gamma_0$. The positive energy wave function of D^0 can be found in paper [18]:

$$\varphi_{P_{f1}}^{++}(\vec{q}) = \frac{M_{f1}}{2}\left(\varphi_1(\vec{q}) + \varphi_2(\vec{q})\frac{m_c + m_q}{\omega_c + \omega_q} \right)$$

$$\times \left[\frac{\omega_c + \omega_q}{m_c + m_q} + \gamma_0 - \frac{\not{q}(m_c - m_q)}{m_q\omega_c + m_c\omega_q} + \frac{\not{q}\gamma_0(\omega_c + \omega_q)}{(m_q\omega_c + m_c\omega_q)} \right]\gamma_5 , \tag{8}$$

where $\omega_c = \sqrt{m_c^2 + \vec{q}^2}$ and $\omega_q = \sqrt{m_q^2 + \vec{q}^2}$; $\varphi_1(\vec{q})$, $\varphi_2(\vec{q})$ are the radial part wave functions, and their numerical values can be obtained by solving the full Salpter equation of 0^- state.

Since the wave function $\varphi_{P_i}^{++}(\vec{q})$ directly relates to the first radially excited state, there is only one node in it. We note that the node structure can possibly play an important role to explain both the narrow decay width and the dominance of $D_s^+\eta$ decay mode, to a certain degree. The value of the radial wave function of 2^3S_1 state becomes negative as a function of relative momentum $\vec{q}$ when it crosses the node (denoted as q_0), that is when $|\vec{q}| > q_0$. Therefore, this negative part of the wave function tends to cancel the contribution to the decay width from the positive part of the same wave function, which can be the reason to have a small decay width. In the overlapping integral of the wave functions Eq. (7) there is the shift of momentum, $\vec{q} - \frac{m_c}{m_c + m_q}\vec{r}$, and this will even strengthen the negative contribution because the shift will move the peak of the radial wave function of the final D meson close to the node. We note that the value of the overlapping integral will be significantly affected by the momentum shift as the larger momentum shift will cause the

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larger negative contribution, and in turn the smaller overlapping integral (smaller decay width). Since the recoil momentum $|\vec{r}|$ of D meson is larger than that of D_s, about 1.43 times, and the momentum shift in case of DK is 1.56 times larger than that of the $D_s\eta$ channel, larger negative contribution results in DK channel than in the $D_s\eta$ channel, which can be the very reason for the small ratio of the branching ratios $DK/D_s\eta$.

As already pointed out in Ref. [3], there may be the $S - D$ mixing in 1^- state [3, 19], so we will try to estimate the possible mixing contributions here too. We first assume that this 1^- state is a pure S wave state 2^3S_1, and later we will consider the case of D wave state. The wave function of 1^- can be written as [19]:

$$\varphi_{P_i}(\vec{q}) = M \left\{ (\psi_1 + \psi_2\gamma_0)\not{\epsilon} + \gamma_0 \left[\gamma_0\vec{q}\cdot\epsilon\psi_1 + (\not{\epsilon}\not{q} - \vec{q}\cdot\epsilon)\psi_2\right] \frac{(\omega_c\omega_q + \vec{q}^2 - m_c m_q)}{(m_c + m_q)\vec{q}^2} \right.$$

$$\left. - \left[\gamma_0\vec{q}\cdot\epsilon\psi_2 + (\not{\epsilon}\not{q} - \vec{q}\cdot\epsilon)\psi_1\right] \frac{(m_q\omega_c - m_c\omega_q)}{(\omega_c + \omega_q)\vec{q}^2} \right\}, \tag{9}$$

and the positive energy wave function is defined as:

$$\varphi_{P_i}^{++}(\vec{q}) = \Lambda_c^+\gamma_0\varphi_{P_i}(\vec{q})\gamma_0\Lambda_q^+ = \frac{1}{2\omega_c}(\omega_c\gamma_0 + m_c + \not{q})\gamma_0\varphi_{P_i}(\vec{q})\gamma_0\frac{1}{2\omega_q}(\omega_q\gamma_0 - m_q - \not{q}), \tag{10}$$

where ϵ is the polarization vector of a $^3S_1$ state; Λ_c^+, Λ_q^+ are the energy projection operators for quark and antiquark; the numerical value of the radial part wave function $\psi_1(\vec{q})$, $\psi_2(\vec{q})$ will be obtained by solving the exact Salpter equation of 1^- state.

IV. NUMERICAL RESULTS AND DISCUSSIONS

For our numerical calculations we use the following parameters: $m_c = 1755.3$ MeV, $m_s = 487$ MeV, $m_d = 311$ MeV, $m_u = 305$ MeV [18], $f_\pi = 130.7$ MeV $f_{\eta_8} = 1.26f_\pi$ MeV, $f_{\eta_0} = 1.07f_\pi$ MeV [23] and $f_K = 159.8$ MeV [24]. The mass of octet η_8 can be estimated by the Gell-Mann-Okubo relation $3m_{\eta_8}^2 = 4m_K^2 - m_\pi^2$, the mass of singlet η_0 can be obtained by $m_{\eta_0}^2 = m_\eta^2 + m_{\eta'}^2 - m_{\eta_8}^2$, and the mixing angle can be estimated by the relation $m_{\eta_0}^2 = \sin^2\theta m_\eta^2 + \cos^2\theta m_{\eta'}^2$. [So our input values are $m_{\eta_8} = 564.3$ MeV, $m_{\eta_0} = 948.1$ MeV, $\theta = -9.95°$.]

We estimate the mass of the first radial excitation of $D_s^*(2112)$ by solving the full Salpeter equation of 1^- state, and our prediction is 2658 ± 15 MeV, which is 26 MeV heavier than the central value 2632 ± 1.7 MeV measured by SELEX. The theoretical uncertainties are estimated by varying all the input parameters simultaneously within $\pm 5\%$, and we choose

the largest possible error. We obtained the wave functions of the corresponding states, and we show radial part wave functions φ_1, φ_2 of D_s in Figure 2; φ_1, φ_2 of D^0 in Figure 3; ψ_1, ψ_2 of 2^3S_1 in Figure 4. We show the overlapping part of 2^3S_1 wave function $\psi_1(|\vec{q}|)$ and D_s wave function $\varphi_1(|\vec{q}| - \frac{m_c}{m_c+m_s}|\vec{r}_{D_s}|)$, when it has the biggest momentum shift $\frac{m_c}{m_c+m_s}|\vec{r}_{D_s}|$ in Figure 5. We also show $\psi_1(|\vec{q}|)$ and D^0 wave function $\varphi_1(|\vec{q}| - \frac{m_c}{m_c+m_u}|\vec{r}_{D^0}|)$ in Figure 6. One can see that in 2^3S_1 wave function the node localizes at $q_0 = 0.54$ MeV, and the biggest momentum shifts are $\frac{m_c}{m_c+m_u}|\vec{r}_{D^0}| = 0.447$ MeV for D^0, and $\frac{m_c}{m_c+m_s}|\vec{r}_{D_s}| = 0.283$ MeV for D_s^+.

The decay width for the two body final state can be written as:

$$\Gamma = \frac{1}{8\pi}\frac{|\vec{r}|}{M^2}|T|^2. \tag{11}$$

Here the matrix element T for $D_{sJ}^+ \to D^0 K^+$ and $D_{sJ}^+ \to D^+ K^0$ is

$$T = \frac{P_{f2}^\mu}{f_{P_{f2}}}\langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle, \tag{12}$$

and for $D_{sJ}^+ \to D_s^+\eta$ it is

$$T = P_{f2}^\mu\left[\frac{-2M_\eta^2\cos\theta}{\sqrt{6}M_{\eta_8}^2 f_{\eta_8}} + \frac{M_\eta^2\sin\theta}{\sqrt{3}M_{\eta_0}^2 f_{\eta_0}}\right]\langle D(P_{f1})|\bar{q}\gamma_\mu\gamma_5 s|D_{sJ}^+(P_i)\rangle. \tag{13}$$

We obtain the following decay widths:

$$\Gamma(D_{sJ}^+ \to D_s^+\eta) = 4.07 \pm 0.34 \text{ MeV}, \tag{14}$$

$$\Gamma(D_{sJ}^+ \to D^0 K^+) = 8.87 \pm 1.23 \text{ MeV}, \tag{15}$$

$$\Gamma(D_{sJ}^+ \to D^+ K^0) = 8.74 \pm 1.22 \text{ MeV}. \tag{16}$$

And the corresponding ratios are

$$\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+\eta) \simeq \Gamma(D_{sJ}^+ \to D^+ K^0)/\Gamma(D_{sJ}^+ \to D_s^+\eta) \simeq 2.2 \pm 0.2. \tag{17}$$

As can be seen, we cannot find the dominance of $D_s\eta$, and the ratio is much different from the experimental ratio 0.14 ± 0.06. The sum of those three decay widths is 21.7 MeV, already larger than the observed bound for full width (≤ 17 MeV) given by SELEX experiment. Therefore, our assumption of the pure S wave vector state cannot fit the experimental data.

Before considering the $S - D$ mixing in 1^- vector state, let us first assume $D_{sJ}^+(2632)$ to be a pure D wave state. In this case, we do not solve the full Salpeter equation, but instead,
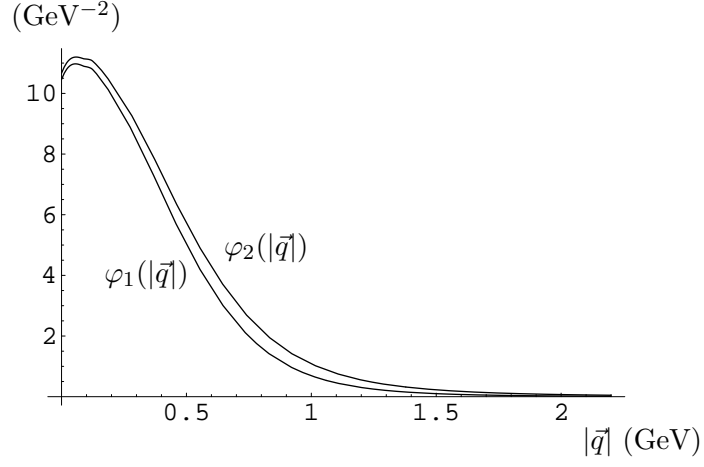
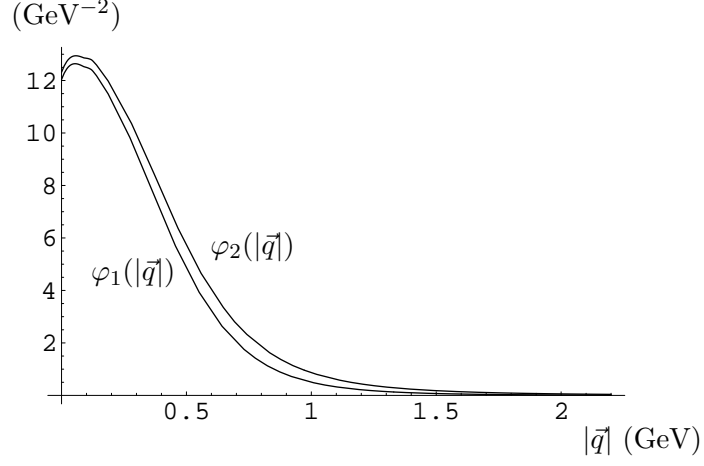FIG. 2: Radial wave function $\varphi_1(|\vec{q}|)$ and $\varphi_2(|\vec{q}|)$ of D_s^+.

FIG. 3: Radial wave function $\varphi_1(|\vec{q}|)$ and $\varphi_2(|\vec{q}|)$ of D^0.
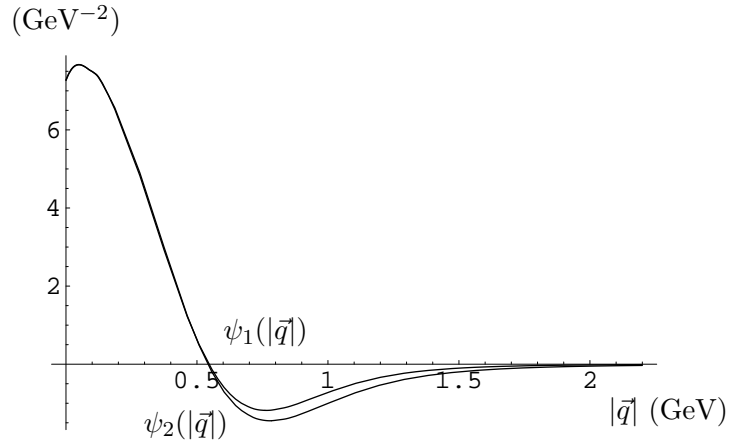

FIG. 4: Radial wave function $\psi_1(|\vec{q}|)$ and $\psi_2(|\vec{q}|)$ of 2^3S_1.
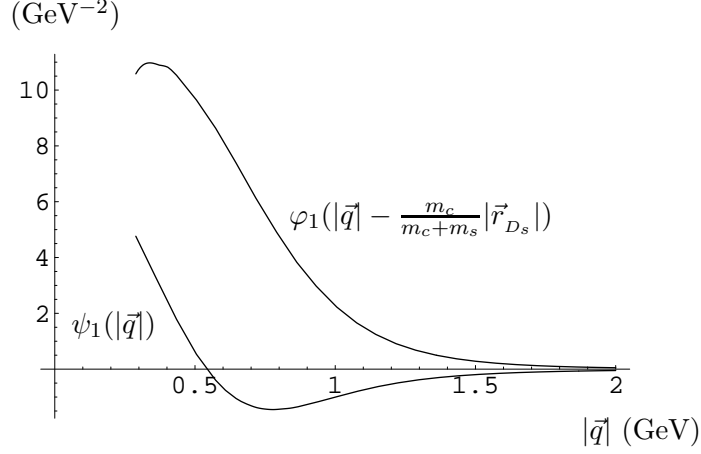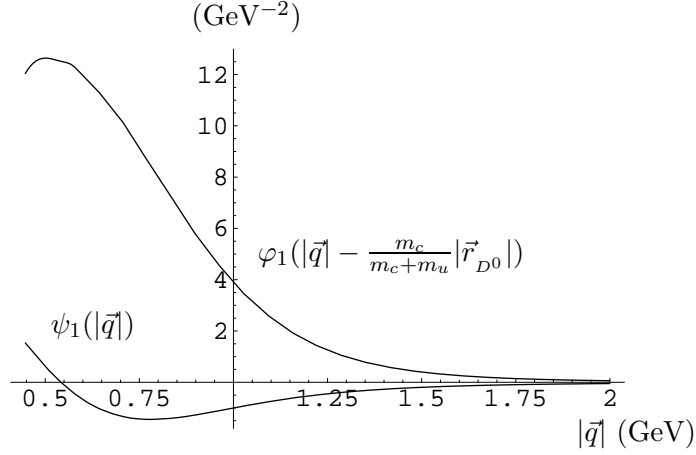
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FIG. 5: The overlapping part of radial wave function $\psi_1(|\vec{q}|)$ and $\varphi_1(|\vec{q}| - \frac{m_c}{m_c+m_s}|\vec{r}_{D_s}|)$ when the momentum of φ_1 has the biggest shift $\frac{m_c}{m_c+m_s}|\vec{r}_{D_s}|$.

FIG. 6: The overlapping part of radial wave function $\psi_1(|\vec{q}|)$ and $\varphi_1(|\vec{q}| - \frac{m_c}{m_c+m_u}|\vec{r}_{D^0}|)$ when the momentum of φ_1 has the biggest shift $\frac{m_c}{m_c+m_u}|\vec{r}_{D^0}|$.

we give the leading order estimate for the pure D wave explanation. The wave function of a pure D wave state $(S - L$ coupling) can be simplified as:

$$\varphi_{P_i}(\vec{q}) = \sum_{s_z, l_z} M(1 + \gamma_0)\not{\epsilon}(s_z)\psi_{nll_z}(\vec{q}) < 1s_z; ll_z \mid 1j_z >, \qquad (18)$$

where $l = 2$. With this wave function as input and by using the same parameters as for the S wave state, we can solve the approximate Salpeter equation, $i.e.$, only the positive energy part solution is considered. The mass of this pure D wave state is 2672 MeV, and the radial wave function of this pure D wave state is shown in Figure 7.

To estimate the matrix element in this case, we calculate the overlapping integral of initial
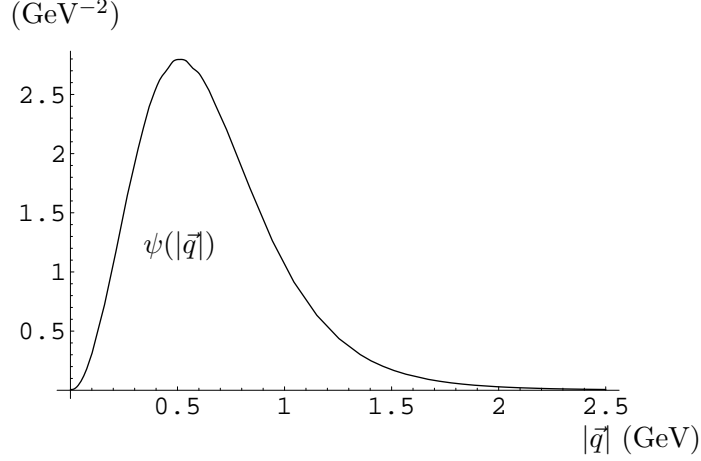
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FIG. 7: Radial wave function $\psi(|\vec{q}|)$ of $D_{sJ}^{+}(2632)$ if it is a D wave state.

and final wave functions. There is only one term, which can give a non-zero contribution:

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\varphi_i(\vec{q}-\frac{m_c}{m_c+m_q}\vec{r})\psi_{nll_z}(\vec{q})q^{\alpha}q^{\beta}f(\vec{q}^2) = \epsilon^{\alpha\beta}(l_z)c, \qquad (19)$$

where $i = 1, 2$; c is a constant; $f(\vec{q}^2)$ is a shorthand notation for the terms which are functions of $\vec{q}^2$; $\epsilon^{\alpha\beta}(l_z)$ is the polarization tensor, which couples with the polarization vector $\epsilon^{\gamma}(s_z)$ to make the total polarization vector $\epsilon^{\alpha}(j_z)$:

$$\sum_{s_z,l_z} \epsilon^{\alpha}(s_z)\epsilon^{\beta\gamma}(l_z) < 1s_z; ll_z \mid 1j_z > = -\sqrt{\frac{3}{20}}\left[\frac{2}{3}\epsilon^{\alpha}(j_z)(-g^{\beta\gamma}+\frac{P^{\beta}P^{\gamma}}{M^2})\right.$$

$$\left. - \epsilon^{\beta}(j_z)(-g^{\alpha\gamma}+\frac{P^{\alpha}P^{\gamma}}{M^2}) - \epsilon^{\gamma}(j_z)(-g^{\alpha\beta}+\frac{P^{\alpha}P^{\beta}}{M^2})\right]. \qquad (l = 2) \qquad (20)$$

Then, we obtain the numerical values for the decay widths when $D_{sJ}^{+}(2632)$ is a pure D wave state ($J^P = 1^-$):

$$\Gamma(D_{sJ}^{+} \rightarrow D_s^{+}\eta) = 3.88 \pm 0.32 \text{ MeV}, \qquad (21)$$

$$\Gamma(D_{sJ}^{+} \rightarrow D^0 K^{+}) = 23.3 \pm 3.2 \text{ MeV}, \qquad (22)$$

$$\Gamma(D_{sJ}^{+} \rightarrow D^+ K^0) = 21.5 \pm 3.1 \text{ MeV}. \qquad (23)$$

And the corresponding ratios are

$$\Gamma(D_{sJ}^{+} \rightarrow D^0 K^{+})/\Gamma(D_{sJ}^{+} \rightarrow D_s^{+}\eta) \simeq \Gamma(D_{sJ}^{+} \rightarrow D^+ K^0)/\Gamma(D_{sJ}^{+} \rightarrow D_s^{+}\eta) \simeq 6.0 \pm 0.5, \quad (24)$$

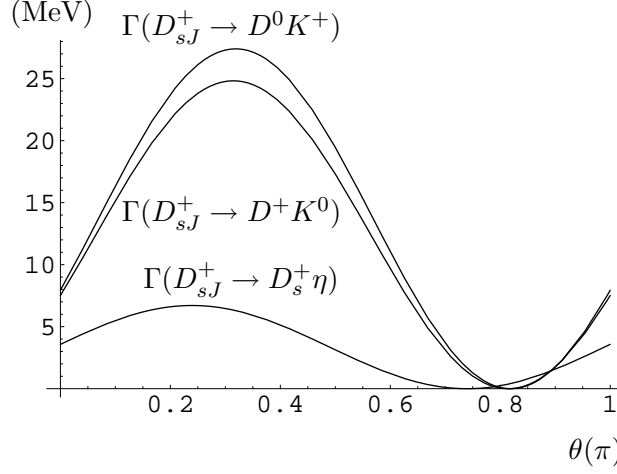which show that the pure D wave assumption does not fit the experimental data obviously.

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FIG. 8: The decay width with different mixing angle θ, when initial state mass is 2632 MeV and it is an $S - D$ mixing state.

We now consider the possibility of the $S - D$ mixing with a mixing angle θ:

$$
\begin{aligned}
|D^*_{sJ}\rangle &= \cos\theta|2S\rangle + \sin\theta|1D\rangle , \\
|D^{**}_{sJ}\rangle &= -\sin\theta|2S\rangle + \cos\theta|1D\rangle .
\end{aligned}
\tag{25}
$$

We show the decay widths for the channels $D^+_{sJ} \to D^+_s\eta$, $D^+_{sJ} \to D^0K^+$ and $D^+_{sJ} \to D^+K^0$ as a function of the mixing angle θ in Figure 8. Numerically it is easy to find that when the mixing angle is taken to be $\theta = 0.807\pi$, the relative branching ratio

$$
\Gamma(D^+_{sJ} \to D^0K^+)/\Gamma(D^+_{sJ} \to D^+_s\eta) = 0.14
\tag{26}
$$

can fit the SELEX data. We also predict other relative branching ratios

$$
\Gamma(D^+_{sJ} \to D^+K^0)/\Gamma(D^+_{sJ} \to D^+_s\eta) \simeq 0.05,
$$
$$
\text{and} \qquad \Gamma(D^+_{sJ} \to D^+K^0)/\Gamma(D^+_{sJ} \to D^0K^+) \simeq 0.37,
\tag{27}
$$

with a rather small total decay width

$$
\Gamma = \Gamma(D^+_{sJ} \to D^+_s\eta) + \Gamma(D^+_{sJ} \to D^0K^+) + \Gamma(D^+_{sJ} \to D^+K^0) + \cdots \simeq 0.35 \ \text{MeV},
\tag{28}
$$

which is quite smaller than the experimental bound for the full width (≤ 17 MeV). It is interesting to note that in fact there is another solution for the mixing angle, $\theta = 0.837\pi$, which can also fit the data of the relative branching ratio. For this solution, accordingly, the relative branching ratios $\Gamma(D^+_{sJ} \to D^+K^0)/\Gamma(D^+_{sJ} \to D^+_s\eta) \simeq 0.20$ and $\Gamma(D^+_{sJ} \to D^+K^0)/\Gamma(D^+_{sJ} \to D^0K^+) \simeq 1.4$ are predicted with a slightly larger total decay width $\Gamma \simeq 0.82$ MeV. The two

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solutions present a different relative aspect on the two decay channels $D_{sJ}^+ \to D^0 K^+$ and $D_{sJ}^+ \to D^+ K^0$.

As a final note, we only estimated the OZI allowed strong decay widths $D_{sJ}^+ \to D_s \eta$ and $D_{sJ}^+ \to DK$ assuming that D_{sJ}^+ is an 1^- vector state. However, there are some other mechanisms through which the 1^- vector state can decay to the same final states, which we do not consider here. For example, there is the OZI forbidden mechanism for D_{sJ}^+ decaying to $D_s \eta$ through the anomaly term in PCAC, $i.e.$ it can decay through two gluons, $D_{sJ}^+ \to D_s + gg$, $gg \to \eta$, which is a decay similar to $\Psi' \to J/\Psi \eta$ (with decay width about 8.9 keV). As is well known, the OZI forbidden processes are small and cannot improve sufficiently the relative ratio to agree with the SELEX data. Therefore, we do not include the processes in our analysis.

As summary, according to the data from SELEX and our estimates made here, it is not likely that the $D_{sJ}^+(2632)$ is the pure S-wave 1^- vector state. Only the $S-D$ wave mixing assumption may explain the relative branching ratio $\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+ \eta) = 0.14$, whereas the assumption seems to give a rather small total decay width (to be compared with the experimental bound $\Gamma \leq 17$ MeV). Generally speaking, our results (based on PCAC and low energy theorem) are qualitatively similar to those of Ref.[3] (based on $^3P_0$ model). We find that the node structure for the first radially excited state of the 1^- ground state $D_s^*(2112)$ can have a favorable effect on the relative ratio of the decay widths but it can not be so big as indicated by the data. The $S-D$ wave mixing may help to explain the data, however, more precise measurements on the relative ratio $\frac{\Gamma(D_{sJ}^+ \to D^+ K^0)}{\Gamma(D_{sJ}^+ \to D_s^+ \eta)}$ (and/or $\frac{\Gamma(D_{sJ}^+ \to D^+ K^0)}{\Gamma(D_{sJ}^+ \to D^0 K^+)}$) as well as the total decay width are crucial. Therefore, the explanation of $D_{sJ}^+(2632)$ as the first radially excited state of the 1^- ground state $D_s^*(2112)$ is too early to conclude.

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